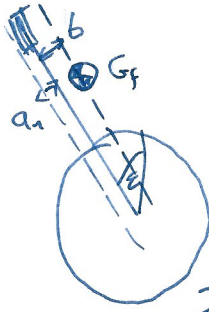




EXAMINATION SOLUTIONS : SESSION 2015/16

Programme:	BEng Motorcycle Engineering	Year:	Level 6
Module/Unit:	Advanced Motorcycle Dynamics		
Author:	Dr Owen Williams	Page:	1 of 3

Question No	Question	Marks
①	Wobble is an oscillation of the front assembly around the steer axis that can become unstable at fairly low to mid speeds. Frequency rises as the trail of the motorcycle increases and as the front frame inertia decreases.  $I_A = I_G + M_f b^2$ Equate moments. $I_G = \text{inertia about CG of } G$ $I_A = \text{inertia about steer axis}$ $c = \text{damping}$ $f_s = \text{lateral tyre force}$ $\delta = \text{steer angle}$ $I_A \ddot{\delta} + c \dot{\delta} + f_s a_n = M_s g b \delta \sin \epsilon + K_{\delta} a_n \delta \sin \epsilon$ = 0 Small $f_s = K_{\delta} \lambda + \dots$ $\lambda_f = \frac{\dot{\delta} a_n}{V} + \delta \cos \epsilon$ $I_A \ddot{\delta} + c \dot{\delta} + f_s a_n = 0$ $\Rightarrow I_A \ddot{\delta} + c \dot{\delta} + K_{\delta} \left(\frac{\dot{\delta} a_n}{V} + \delta \cos \epsilon \right) a_n = 0$ $\delta = e^{st}$ $\Rightarrow I_A s^2 + \left(c + \frac{K_{\delta} a_n^2}{V} \right) s + K_{\delta} a_n \cos \epsilon = 0$ quadratic formula $\Rightarrow s = \frac{-cV + K_{\delta} a_n^2}{2I_A V} \pm \sqrt{\left(\frac{cV + K_{\delta} a_n^2}{2I_A V} \right)^2 - \frac{K_{\delta} a_n \cos \epsilon}{I_A}}$ Complex when discriminant is negative.	15

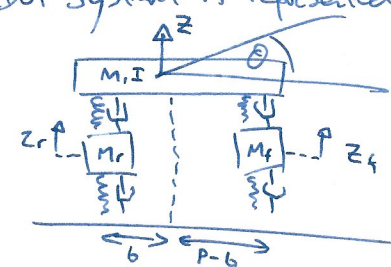
Continuation Sheet

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Question No	Question	Marks
2	Gyro question, so $G_m = I \omega \dot{\theta} \Rightarrow G_m = I_f \omega_f \dot{\theta}$ Steer rate determined from wobble frequency $\Rightarrow 88.2^\circ/s$ $\omega_f = 49.3 \text{ rad/s} \Rightarrow 1.54 \text{ rad/s}$ So $G_m = 0.42 \times 49.3 \times 1.54 \times \cos 42$ $G_m = 23.7 \text{ Nm}$ The oscillation is about the steer axis so variations in steer angle are seen at wobble frequency resulting in instability Changes in roll angle may be seen as a result with $G_m = I_x \omega_f \dot{\theta} \cos \theta$ $\dot{\theta} = 0.021 \text{ rad/sec} \Rightarrow 1.22^\circ/\text{sec}$	
3a	Neglecting engine contributions $G_m = \cancel{(I_f \times \omega_f)} (I_f \times \omega_f) + (I_r \times \omega_r) \omega \cos \theta$ $= (0.42 \times 29.1) + (0.55 \times 42.1) \times 1.8 \times 1$ $= \underline{63.7 \text{ Nm}}$	<u>5</u>
b,	force required to control motion $\rightarrow$ landing is the force at the bars to overcome the torque, so in this case Simply <u>245 N</u> at the throttle hand or equivalent.	<u>5</u>

Continuation Sheet

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Question No	Question	Marks
4a	This 4 Dof system is represented as:  Half car model Springs and Dampers Resolving forces for undamped system. $M\ddot{z} + (K_f + K_r)z + (P-b)K_f\theta - bK_r\theta - K_f z_f - K_r z_r = 0$ $\Rightarrow M\ddot{z} + (K_f + K_r)z + ((P-b)K_f - bK_r)\theta - K_f z_f - K_r z_r = 0$ Rotationally $I\ddot{\theta} + ((P-b)K_f - bK_r)z + ((P-b)^2 K_f + b^2 K_r)\theta - (P-b)K_f z_f + bK_r z_r = 0$ for M_f: $M_f \ddot{z}_f - K_f z - (P-b)K_f \theta + (K_f + K_{tf})z_f = 0$ and $M_R \ddot{z}_r - K_r z + bK_r \theta + (K_r + K_{tr})z_r = 0$ This gives $\Rightarrow$ $\begin{bmatrix} M & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & M_f & 0 \\ 0 & 0 & 0 & M_r \end{bmatrix} \begin{bmatrix} \ddot{z} \\ \ddot{\theta} \\ \ddot{z}_f \\ \ddot{z}_r \end{bmatrix} + \begin{bmatrix} K_f + K_r & (P-b)K_f - bK_r & -K_f & -K_r \\ (P-b)K_f - bK_r & (P-b)^2 K_f + b^2 K_r & -(P-b)K_f & bK_r \\ -K_f & -(P-b)K_f & K_f + K_{tf} & 0 \\ -K_r & bK_r & 0 & K_r + K_{tr} \end{bmatrix} \begin{bmatrix} z \\ \theta \\ z_f \\ z_r \end{bmatrix} = 0$ Damping can be added by simple $K \rightarrow C$ substitution so $MA + CV + KX = 0$ This is simply a description of the bicycle steering model low speed, non gyro controlled, balance and instability under constant control, overturning moment $\rightarrow$ counter-steer	15 5 60